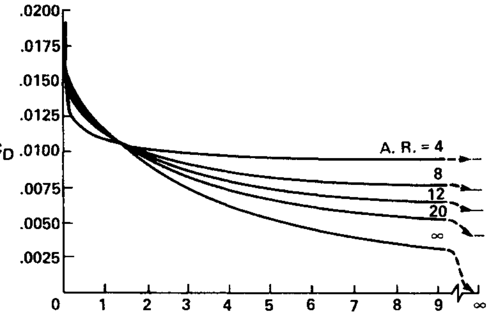

Fig. 3 Inviscid drag coefficient of rectangular plates of varying aspect ratio as a function of nondimensional time.

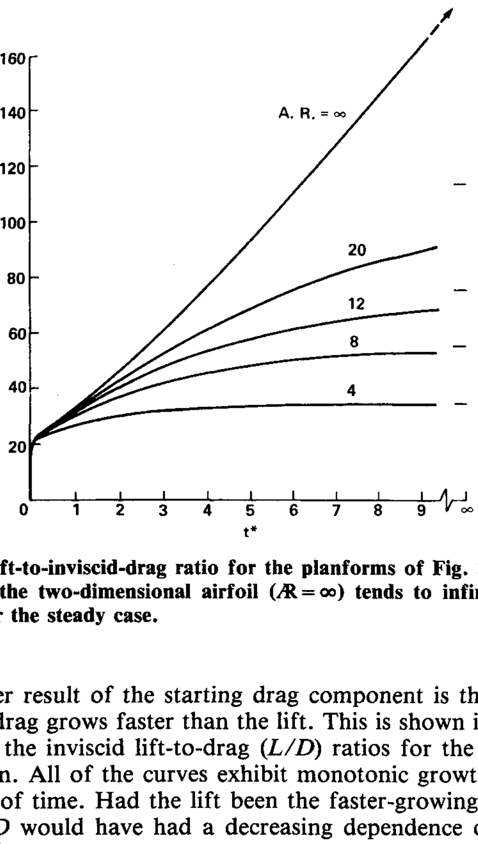

Fig. 4 Lift-to-inviscid-drag ratio for the planforms of Fig. 3. The curve for the two-dimensional airfoil ($AR = \infty$) tends to infinity as $C_{Di} = 0$ for the steady case.

Another result of the starting drag component is that the induced drag grows faster than the lift. This is shown in Fig. 4, where the inviscid lift-to-drag (L/D) ratios for the wings are shown. All of the curves exhibit monotonic growth as a function of time. Had the lift been the faster-growing function, L/D would have had a decreasing dependence on the elapsed time.

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Unified Supersonic/Hypersonic Similitude for Oscillating Wedges and Plane Ogives

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Introduction

THE large deflection hypersonic similitude of Ghosh[1] was applied by Ghosh and Mistry[2] to the case of an oscillating wedge and plane ogive (nonplanar wedge) to obtain a closed-form expression for the pitching moment derivative due to the rate of pitch Cm_q. This derivative had been denoted $Cm_{\dot{\alpha}}$ in Ref. 2 due to a later discovered error in the nomenclature and the similitude extended to oscillating delta wings in Ref. 3. In Ref. 4, a similitude for nonslender cones and quasicones has been outlined and the cylindrically symmetric piston motion of Sedov[5] extended. Hui[6] developed a small-perturbation approach to treat oscillating wedges with attached shocks in supersonic/hypersonic flow. In this Note, a unified supersonic/hypersonic similitude for a wedge and quasiwedge is given. This approach was first reported in abstract form in Ref. 7.

Steady Wedge

Figure 1 shows the upper half of a wedge at constant-zero incidence with an attached bow shock in rectilinear flight (at time t) in stationary inviscid air. The wedge starts its motion at time $t = 0$ from point 0. Dimensional analysis indicates that the flow is conical in nature, i.e., at a given instant $\partial/\partial r = 0$, where r is the distance along a ray from the apex. Therefore, the bow shock must coincide with a ray. The space-fixed coordinate system (x,y) is so chosen that the x axis coincides with the bow shock at time $t = 0$. The conicality of the flow implies that the streamlines at an instant t have the same slope where they intersect a particular ray from the apex. Since the shock sets the fluid particles in a motion normal to itself, the instantaneous streamlines must intersect the shock at right angles. The dashed lines in Fig. 1 are the probable streamline shapes. We tentatively assume that the streamlines are straight as shown by the solid lines in Fig. 1; if this leads to $\partial/\partial r = 0$, then it is a solution. Consider the plane flow on a stream surface $x = 0$. At time t, the shock location on $x = 0$ is

$$y_s = U_\infty t \sin\beta \tag{1a}$$

and it can be shown from geometry that the body location is

$$y_b = OB = U_\infty t \sin\delta / \cos\varphi \tag{1b}$$

Since the inviscid flow in plane $x = 0$ is independent of the flow in a neighboring parallel plane, it can be taken as a piston-driven fluid motion where the piston velocity is dy_b/dt and hence the piston Mach number is

$$M_p = M_\infty \sin\delta / \cos(\beta - \delta) \tag{1c}$$

The shock Mach number in this plane of motion is $M_s = M_\infty \sin\beta$. Since the piston velocity is independent of time, $\partial p/\partial y = 0$. Since the streamlines are straight, there is no centrifugal force on the fluid particles. Hence, $\partial p/\partial x = 0$ and $\partial p/\partial r = 0$. Thus, the wedge flow is exactly equivalent to one-

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dimensional plane piston motion normal to the shock. It can be shown that the one-dimensional relation between M_s and M_p yields the oblique shock relation between β and δ.

Quasiwedge or Oscillating Wedge

Figure 1 shows the probable shape of the bow shock (dashed line) when the wedge is either oscillating or replaced by a quasiwedge. The slope of the curved shock with the x axis remains small, say, of order Φ. We define (x_1, y_1) as a coordinate system fixed to the nose of the wedge (Fig. 1). For order of magnitude analysis, we consider the flow past a steady wedge. Let the Mach number behind the shock in the body-fixed coordinate (x_1, y_1) be M_2. The characteristics make an angle of, say, $E\,[=\sin^{-1}(1/M_2)-\varphi]$ with the x_1 axis. $E \ll 1$ up to fairly large values of δ, even for moderate freestream Mach numbers. For example, $E=16$ deg for $M_\infty=2.5$ and $\delta=22.5$ deg. We impose another constraint that M_∞ and δ are such that

$$E \leq 0.3 \tag{2}$$

and Φ and E are of the same order. Note that the flow is not uniform and that Mach waves are present.

The gradient is normal to the characteristics. Therefore

$$\frac{\partial}{\partial x}=0\left(\Phi\cdot\frac{\partial}{\partial y}\right) \tag{3a}$$

Also, the net perturbation introduced by the shock and the Mach waves (which are of small inclination) will be chiefly in the y direction. Thus,

$$u=0(\Phi\cdot v) \tag{3b}$$

where u and v are the net velocity components in the space-fixed coordinates x and y, respectively. Equations (3) suggest the transformations

$$u'=\Phi^{-1}\cdot u \quad \text{and} \quad x'=\Phi\cdot x \tag{4}$$

Applying these transformations to the equations of motion and boundary conditions and neglecting the terms of $\mathcal{O}(\Phi^2)$, in the same way as in Ref. 4 results in an equivalence having a one-dimensional piston motion in the y direction that is normal to the plane approximating the curved shock.

We choose coordinates (x', y') such that x' is along the wedge surface (Fig. 1). According to hypersonic large-deflection similitude,[2,4] the equivalent piston moves in the y' direction with the Mach number, say M_p'. In the context of the present unified similitude, the piston Mach number for a steady wedge [Eq. (1c)] is

$$M_p=M_p'/\cos\varphi \tag{5}$$

When the wedge upper surface is pitching about pivot point x_0', the surface element at x' has an additional velocity in y' direction equal to $-(x'-x_0')q$, where q is the pitch rate assumed to be positive in the nose-up sense. The adjacent fluid particle, which must have the same velocity normal to the surface, is constrained by the unified similitude to have a net velocity vector parallel to the y direction. Hence, the additional particle velocity v_a that arises due to the pitching motion is also parallel to y. Therefore, the component of v_a in the y' direction is

$$v_a\cos\varphi=-(x'-x_0')q$$

or

$$v_a=-(x'-x_0')q/\cos\varphi$$

The "equivalent" piston has the same velocity as that of a particle adjacent to the surface. Therefore, Eq. (5) is also valid for an oscillating wedge; hence,

$$M_p=\frac{U_\infty\sin(\delta-\theta)-(x'-x_0')q}{(a_\infty\cos\varphi)} \tag{6}$$

where θ is the pitch angle. The motion is assumed to be quasisteady so that pressure p is a function, given in Eq. (2) of Ref. 2, of the instantaneous value of M_p. This approach yields Cm_q. The formulas of Ref. 2 need to be adjusted only for the $\cos\varphi$ factor of Eq. (5). Thus, the derivatives for a wedge are

$$-Cm_\theta=\frac{(\gamma+1)\tan\delta}{\cos^2\varphi}\cdot\left(2+D'+\frac{1}{D'}\right)\left(\frac{1}{2}-h\cos^2\delta\right) \tag{7}$$

$$-Cm_q=\frac{(\gamma+1)\tan\delta}{\cos^2\delta\cdot\cos^2\varphi}\cdot\left(2+D'+\frac{1}{D'}\right)$$

$$\times\left(\frac{1}{3}-h\cos^2\delta+h^2\cos^4\delta\right) \tag{8}$$

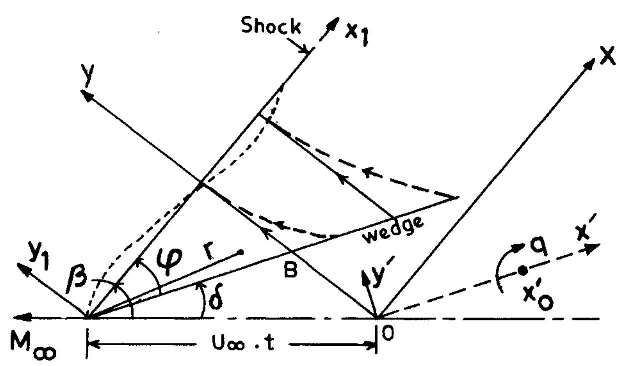

Fig. 1 Wedge at time t: coordinate systems.

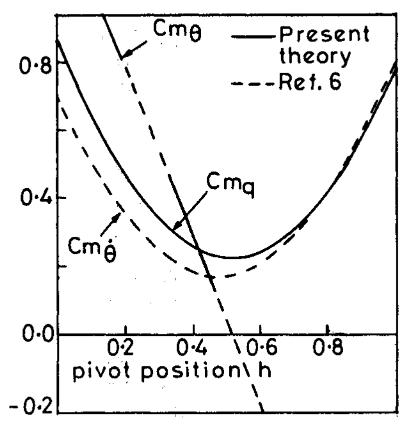

Fig. 2 Derivatives in pitch for a wedge: $M_\infty=3$, $\delta=10$ deg, $\gamma=1.4$.

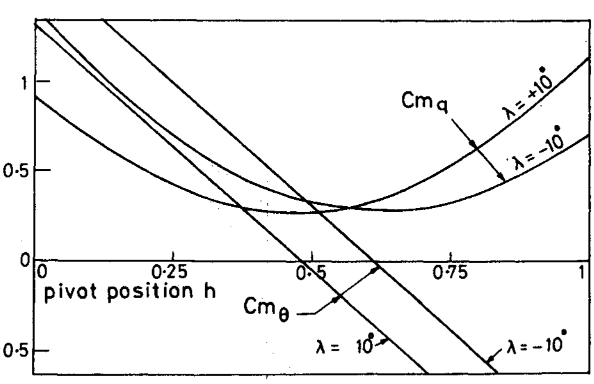

Fig. 3 Derivatives in pitch for plane ogives: $M_\infty=3$, $\delta=15$ deg, $\gamma=1.4$.

where h is the nondimensional pivot position and D' can be obtained by replacing M_∞ by $M_\infty/\cos\varphi$ in D of Ref. 2. Also, allowance has to be made for a different notation, e.g., the wedge half-angle is δ, whereas in Ref. 2 it is θ.

For a plane ogive (the nonplanar wedge of Ref. 2),

$$M_p = \text{right side of Eq. (6)} + \frac{U_\infty \cos(\delta-\theta)\,(\mathrm{d}y'/\mathrm{d}x')}{a_\infty \cos\varphi} \tag{9}$$

where $\mathrm{d}y'/\mathrm{d}x'$ is the slope of the upper surface.

The derivatives for an ogive are

$$-Cm_\theta = \frac{(\gamma+1)}{2M_\infty^2 \cos^2\delta}(I_1' + I_2' + I_3') \tag{10a}$$

$$-Cm_q = \frac{(\gamma+1)}{2M_\infty \cos^3\delta \cos\varphi}(J_1' + J_2' + J_3') \tag{10b}$$

where I_1', J_1', etc. can be obtained from I_1, J_1, etc. of Ref. 2 in the same way as D'.

Discussion and Results

For a steady wedge, the present similitude applies in a plane normal to the shock, is exact, and is restricted to $M_\infty > 1$. The similitude of Ref. 2 applies in a plane normal to the wedge surface, has an error of $\mathcal{O}(\varphi^2)$, and is restricted to $M_\infty \geq 5$. For a quasiwedge or an oscillating wedge, another constraint in addition to the Mach number restriction is necessary: $E \leq 0.3$ for the present similitude and $M_2 \geq 2.5$ for the similitude of Ref. 2 (see also Ref. 4).

In the hypersonic domain, both similitudes are valid and the order-of-magnitude analysis indicates the existance of the same error for both since E, $\Phi = 0(\varphi)$. Also since $\varphi \ll 1$, the cos φ factor of Eq. (5) makes a small difference of $\mathcal{O}(\varphi^2)$, which is within the error of the similitude of Ref. 2. However in the context of the present similitude, the solution for quasiwedge or oscillating wedge is a small departure from an exact solution and, hence, is likely to be an improvment on Ref. 2.

Cm_θ is concerned with a steady rotation in pitch; hence, the present method is exact and gives identical results with Ref. 6 (Fig. 2). Cm_q shows a comparable trend with Cm_θ (Fig. 2); note that there is no other theory for Cm_q. The effect of convexity in the plane ogives is to decrease the stiffness and shift the damping minima forward (Fig. 3).

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Shock/Turbulent Boundary-Layer Interaction with Wall Function Boundary Conditions

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Introduction

In recent years, advances in the efficiency of numerical methods[1,2] for the computation of compressible viscous flows have substantially reduced computing times. The problem of the computation of turbulent flows, however, remains a major challenge. One difficulty is due to the fact that integration of the Reynolds-averaged Navier-Stokes equations up to the wall with zero slip boundary conditions needs fine mesh spacing in the vicinity of the wall in order to capture rapid variation of the flowfield in that region.

Launder and Spalding[3] and Chieng and Launder[4] have demonstrated that the wall function approach eliminates the requirement of very fine mesh close to the wall. Their treatment was, however, limited to incompressible flows. Recently, Rubesin and Viegas[5] have extended the wall function approach to complex compressible flows. In all these attempts, wall functions were used with two equation models of turbulence, and the effect of adverse pressure gradient was not taken into account in the law of the wall.

The present Note describes a formulation of wall function approach to boundary conditions in which a modified law of the wall is employed in the regions where adverse pressure gradient exists. Its effectiveness is demonstrated by application to the problem of numerical simulation of shock/turbulent boundary-layer interaction with an adiabatic wall and Cebeci-Smith model of turbulence. Substantial saving in computation time and memory requirements is achieved without significantly affecting the accuracy of the solution. The formulation can be easily extended to treat a nonadiabatic wall.

Analysis

The differential equations used to describe the mean flow for this study are the time-dependent, Reynolds-averaged Navier-Stokes equations written in conservation form and Cartesian coordinates for plane flow of a compressible fluid.

The present wall function approach is depicted in Fig. 1. The first mesh volume off the surface is between $y=0$ and $y=y_e$ such that its center (2) is located well into the fully turbulent region ($y^+ \geq 50$). This volume is only partly filled with fully turbulent flow and is partially composed of the viscous sublayer with its edge at $y=y_v$. The second mesh volume is between $y=y_e$ and $y=y_f$ and is centered at point (3). Point (1) is the mirror image of point (2) and is the center of the fictitious boundary cell. All these mesh volumes described above are used in the numerical computation. However, in the wall function model, only the first mesh volume off the surface is used.

The incompressible law of the wall is given by

$$u^+ = (1/K)\ln(y^+) + B \tag{1}$$

where

$$K = 0.41 \text{ and } B = 5$$

$$u^+ = u/u_\tau,\quad y^+ = yu_\tau/\nu_\omega,\quad \nu_\omega = \mu_\omega/\rho_\omega,\quad \text{and}\quad u_\tau = (\tau_\omega/\rho_\omega)^{1/2}$$

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